Geometric Transformations and Linear Functions: Two Sides of a Coin

Session 245 2015 NCTM Annual Meeting

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Common Core Standard G-CO2 says that students should "describe transformations as functions that take points in the plane as inputs and give other points as outputs." This statement only hints at the potential impact that this connection has for students' understanding of functions.

Recent findings in brain science and in cognitive science—the science that studies how we think, that analyzes the nature of our knowledge and understanding—support the age-old wisdom that we learn better by doing than by being told. Our bodies are not separate from our brains, and we learn best by exploiting that connection.

For functions, this means that students should experience, in as physical a way as possible, how variables vary, the shape and size of a domain and a range, the actual motion that underlies relative rate of change. These experiences enable students to form *conceptual metaphors* on which to base their abstract mathematical ideas:

> The mechanism by which the abstract is comprehended in terms of the concrete is called *conceptual metaphor*. Mathematical thought … makes use of conceptual metaphor, as when we conceptualize numbers as points on a line. [Lakoff & Nuñez, *Where Mathematics Comes From,* p. 5]

Hans Freudenthal expressed the importance of learning based on our sensory-motor systems this way:

> Geometry is one of the best opportunities that exists to learn how to mathematize reality….[N]umbers are also a realm open to investigation…but discoveries made by one's own eyes and hands are more convincing and surprising. [Freudenthal, *Mathematics as an Educational Task,* p. 407]

In this session we'll literally *transform* students' understanding of linear functions, making their discoveries "more convincing and surprising" by using geometric transformations to connect their eyes and hands specifically to linear functions and more generally to function concepts.

We'll do that by having students create and manipulate geometric transformations, explore them as functions, restrict them to a number line, connect them to multiplication and addition, compose them to produce linear functions, animate their variables, and rotate the result to produce the Cartesian graph of $y = mx + b$.

By treating geometric transformations as functions, students *vary* the variables by dragging them, observe *rate of change* by watching the variables move, use meaningful *function notation,* and forge a strong link between *geometry and algebra.*

Web Sketchpad activities are here: geometricfunctions.org/nctm2015.html

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