Sometimes you want to combine two functions. For instance, you could create an interesting animation by first rotating a picture and then reflecting the rotated picture. When we use two functions in this way, we say that we *compose* the two functions.

In this activity, you will dilate a point on the number line and then translate the result. In the terms of mathematics, you will *compose* a dilate function and a translate function.

DILATE AND TRANSLATE A POINT ON THE NUMBER LINE

1. Open http://geometricfunctions.org/links/compose-on-a-line/ and go to page 2.



- 2. Construct a Number Line , construct Point x on the line, and Dilate x point x
- **Q1** Change scale factor *s*, and drag independent variable *x*, to fill in this table.

Scale factor s	D(x) goes the [same/opposite] direction as x.	D(x) goes [faster/slower] than x.
s = 0.25		
s = -0.25		
s = 2.50		
s = -2.50		



- 3. Tap $\frac{x}{\text{Translate } v}$ and translate D(x) by vector v. The result is T(D(x)), which stands for the *translation* of the *dilation* of x.
- 4. Use the $\frac{2^{x}}{Measure}$ tool three times, to measure x, D(x), and T(D(x)).
- **Q2** For each row of the table, edit *s* and drag *v* and *x* to the values shown. Then record the values for D(x) and T(D(x)).

S	v	X	D(x)	T(D(x))
1.50	-4.00	-2.00		
-0.60	4.00	5.00		
-3.00	-2.50	1.50		

RELATIVE RATE OF CHANGE

The values of *s* and *v* both affect the composed function.

Q3 Do an experiment to see how s and v affect the rate of change of dependent variable T(D(x)) relative to independent variable x. The idea is to figure out exactly how much the dependent variable changes by when the independent variable changes by 1.

Choose friendly values for s and v, and then record the values of x and T(D(x)) as the first location. Drag x exactly 1.00 unit to the right, and record the new values of x and T(D(x)). Record the difference in the last column. (The difference in x-values should always be 1.00.)

scale	vector	start location	1 unit right	difference
	v =	<i>x</i> ₁ =	<i>x</i> ₂ =	1.00
S =		$T(D(x_1)) =$	$T(D(x_2)) =$	
	v =	<i>x</i> ₁ =	<i>x</i> ₂ =	1.00
S =		$T(D(x_1)) =$	$T(D(x_2)) =$	
s =	v =	<i>x</i> ₁ =	$x_2 =$	1.00
		$T(D(x_1)) =$	$T(D(x_2)) =$	

Q4	What did you notice about your results?			
Q5	How can you adjust your function so that $T(D(x))$ will change by exactly 1.50 when x			
	changes by exactly 1.00? Explain.			

ON YOUR OWN



Q6 Use the Mystery function tool to create an unknown mystery function. The dependent variable is shown as ??(x). Your job is to adjust s and v so that no matter how you vary x, your dependent variable T(D(x)) exactly matches ??(x). When you have it, press the Check button to be sure, and then try a new problem.

INDEPENDENT AND DEPENDENT VARIABLES

When you compose two functions, you've combined the two to make a new function, and you should be careful not to get them confused with each other. The table below may help you keep them straight. (In the sketch we use nicknames to avoid cluttering the screen.)

Family	Family name	Full name	Meaning of name	Input	Output	Output Nickname
Dilate	D	$D_{0,s}$	Dilate about center 0 by scale factor <i>s</i>	X	$D_{0,s}(x)$	D(x)
Translate	T	T_{v}	Translate by vector v	$D_{\theta,s}(x)$	$T_{v}\left(D_{\theta,s}(x)\right)$	T(D(x))
Composed	$T \circ D$	$T_{v} \circ D_{0,s}$	Translate by <i>v</i> after dilating about <i>0</i> by <i>s</i>	X	$T_{v}\left(D_{\theta,s}(x)\right)$	T(D(x))