Notes for the Teacher

Students use grids of virtual algebra tiles to interpret and solve binomial multiplication problems. They focus on problems of the form $(ax + b)(cx + d)$, where b and/or d is negative. Students determine the dimensions of the grid and then add the partial products to find the total product.

Note: If possible, start with the activity **Binomial Multiplication Part One—Dynamic Algebra Tiles**. It introduces the algebra tile model used in this activity.

Objectives:

- Students will find the products of two binomials using a grid of algebra tiles.
- Students will use virtual algebra tiles to expand binomial multiplication problems of the form $(ax + b)(cx + d)$, where b and/or d is negative.
- Students will relate a multiplication problem represented in numerical form to the same problem represented with algebra tiles.

Common Core Mathematical Practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (5) Use appropriate tools strategically; (7) Look for and make use of structure.

Common Core State Content Standards: A-APR; A-SSE3a

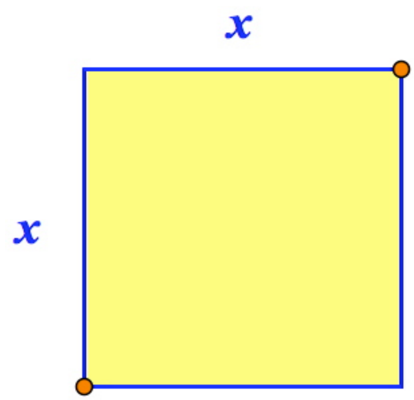
Grade Range: Grades 8–9

Introduce:

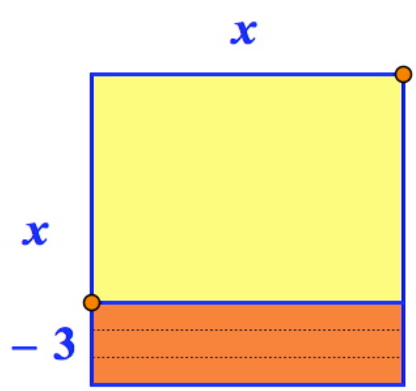
Open **Binomial Multiplication Part Two--Dynamic Algebra Tiles.gsp** and distribute the worksheet. Use a projector to show page “Grid 1.” Ask, “What multiplication problem is represented by this model?” The model shows $(x + 3)(x + 7)$ with algebra tiles. Briefly review how to compute the product using the four partial products corresponding to the algebra tiles: $x^2 + 7x + 3x + 21 = x^2 + 10x + 21$.

Explain, “Let’s explore how we can represent a binomial multiplication problem like $(x – 3)(x – 7)$ that has negative terms. We’ll start with $x \cdot x$.”

Drag the orange points with the **Arrow** tool so that the grid shows only $x \cdot x$. Ask, “What is the area of this grid?” ($x \cdot x = x^2$)



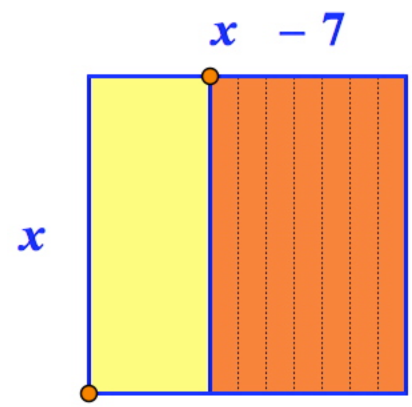
Say, "Now let's change the problem from $x \cdot x$ to $(x - 3) \cdot x$. Let's see how we can represent the multiplier $(x - 3)$." Drag the orange point in the lower-left corner up 3 units to show a side length of $x - 3$. Doing so creates an orange region in the grid, as shown below.



Ask, "What are the dimensions of the grid now?" [$(x - 3) \cdot x$] Continue, "The area of the original yellow grid with dimensions $x \cdot x$ was x^2. What should we do to find the area of this new grid with dimensions $(x - 3) \cdot x$?" Students may reply that they need to subtract the area of the orange grid from the area of the original yellow grid.

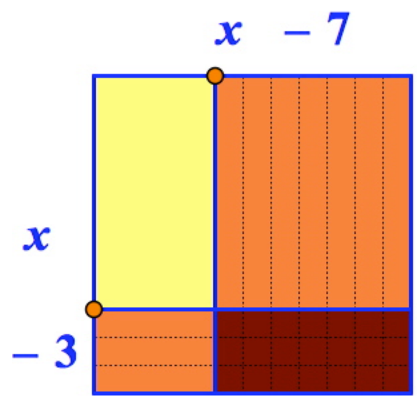
Ask, "What is the area of the orange grid?" (Since the dimensions of the orange grid are 3 and x, its area is $3x$.) "Remember the area of the original yellow grid was $x \cdot x = x^2$. So what is the area of $(x - 3) \cdot x$? Explain." ($x^2 - 3x$ because you have to subtract the area of the orange grid from the area of original yellow grid.)

Say, "We could have represented the factor $(x - 7)$ first." Drag the orange point to return the grid to its original x by x dimensions. Ask a volunteer to demonstrate subtracting 7 from x by dragging the orange point in the top-right corner. Doing so creates the picture below:



Ask, "What is the area of this orange grid?" ($x \cdot 7 = 7x$) "Remember the area of the original yellow grid was $x \cdot x = x^2$. So what is the area of $x \cdot (x - 7)$? Explain." ($x^2 - 7x$ because you have to subtract the area of the orange grid from the area of the original yellow grid.)

Say, "Let's see what happens if we model both factors, $x - 3$ and $x - 7$, at the same time. (Drag the lower-left point to show $x - 3$ again.)



Ask, "How does this grid represent $(x - 3)(x - 7)$?" Students may reply that the dimensions of the large grid are $(x - 3)$ by $(x - 7)$.

Remind students that the area of the $x \cdot 7$ grid was $7x$ and the area of the $3 \cdot x$ grid was $3x$. Say, "Maybe if we subtract the areas of both orange grids from the area of the $x \cdot x$ grid, we'll find the area of the $(x - 3)(x - 7)$ grid. Let's try that."

Write the following on the board, asking students to complete each calculation:

Area of the original x by x yellow grid: x^2

Combined orange areas: $7x + 3x = 10x$

Area of the yellow grid – Area of the orange grids = $x^2 - 10x$

Does $(x - 3)(x - 7) = x^2 - 10x$?

Elicit that they are not equal. Give students time to examine the model and think about why the subtraction strategy does not work. Students will likely notice that the two orange grids overlap. The overlap is represented by the 3 • 7 grid, or 21 ones, in the lower-right corner of the overall grid. These ones are shaded more darkly because they represent the overlapping area.

It is important to recognize that the 21 ones in the lower-right corner of the overall grid were subtracted twice: first as part of the $(x - 3) \cdot x$ grid and again as part of the $x \cdot (x - 7)$ grid. To correct for this overcount, 21 must be added back to $x^2 - 10x$ to obtain the correct product of $(x - 3)(x - 7)$, which is $x^2 - 10x + 21$.

This chain of reasoning is by no means simple. Spend time modeling other binomial multiplication problems that involve subtraction. A visual pattern will emerge: The grid always begins as yellow, with dimensions x by x and area x^2. Two orange grids are then subtracted from the yellow grid. The area of the new yellow grid will be equal to the area of the original yellow grid minus the areas of the orange grids. But because the area of the overlapping region was subtracted twice as part of both orange grids, this area needs to be *added back* to the other partial products. [For your own reference, this can be stated algebraically as $(x - a) \times (x - b) = x^2 - ax - bx + ab$.]

The partial products may be viewed by pressing the *Show Partial Products* button with the **Arrow** tool.

Explore:

Assign students to partners and send them in pairs to the computers. Have students open **Binomial Multiplication Part Two--Dynamic Algebra Tiles.gsp** and go to page “Grid 1.” Ask students to use the model to find the products of the binomial multiplication problems. Make sure students understand how to record the results on the worksheet. Explain that students should sketch the grids, write the partial products, and find their sum.

After solving a few problems, students will move on to page “Grid 2.” Here students will solve binomial multiplication problems in which the x coefficients of both binomials are greater than 1. Watch for students who may have difficulty applying what they have done previously to offer extra guidance.

As you circulate, observe students as they work. Do students understand why they are subtracting the areas of the orange grids? Do students understand why the overlapping, darkly shaded region must be added back? Stop and ask several students to explain their reasoning.

Discuss:

Call students together to discuss the worksheet. Open **Binomial Multiplication Part Two--Dynamic Algebra Tiles.gsp** and go to page "Grid 1" and later to "Grid 2."Ask a few volunteers to model their solutions. Then break their solution methods down and have students explain why certain areas are being subtracted and others added. Ask questions like the following:

- *How does your model represent the multiplication problem?*
- *Why did you represent the first factor this way? The second factor?*
- *What do the orange grids show?*
- *What does the overlapping region mean and what do you do about it?*
- *What was different about the problems that you solved using the model on page "Grid 2"?*
- *Do you notice any patterns in the answers?*

Answers:

2. $(x - 2)(x + 2) = x^2 - 4$
3. $(x - 5)(x - 4) = x^2 - 9x + 20$
4. $(x - 8)(x - 3) = x^2 - 11x + 24$
5. $(2x + 1)(3x - 2) = 6x^2 - x + 2$
6. $(5x - 5)(4x - 3) = 20x^2 - 35x + 15$
7. $(3x - 8)(3x - 8) = 9x^2 - 48x + 64$

Related Activities:

- *Binomial Multiplication, Part One—Dynamic Algebra Tiles*
- *Factoring Games, Part One—Dynamic Algebra Tiles*
- *Factoring Games, Part Two—Developing Factoring Fluency*

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