Use this activity as an introduction to integer addition for pre-algebra students, as a start-of-the-year refresher for Algebra 1 students, or as a supplemental activity for any student having difficulty with the topic. It's important for students to have a mental image of operations on integers. Even strong students who rely on verbal rules make careless mistakes that could be avoided by having an internalized picture.

The picture of addition presented here is a geometric model in which each number is represented by a vector. (The activity calls them *arrows* because students may not be familiar with the term *vector*.) Vectors incorporate both magnitude and direction (representing the absolute value and the sign of the integer), so practice with this model helps students understand how the signs of the addends come into play.

This activity contains lots of questions for students, who develop their understanding through the process of manipulating the sketch and describing what they observe. Encourage them to write clear and detailed explanations (and to use complete sentences) when they answer the questions; the extra time it takes them to do so is well spent.

If there's time and you have a presentation computer with a projector, have different students use Sketchpad to demonstrate to the class their observations or the problems they made up. It's a big help to students if they can listen to, evaluate, and discuss the descriptions and conclusions of their classmates.

INVESTIGATE

Students may be unfamiliar with *model* as a transitive verb; consider reviewing with them the various uses of this word.

Q1 In their final positions, the second arrow starts from where the first arrow ends, and the answer (13) is at the end of the second arrow. Encourage students to be detailed and specific in their answer to this question.

Q2 Answers will vary but should include only positive numbers.

Q3 Each lower arrow is exactly the same size and direction as the corresponding upper arrow.

Q4 The sum of $-6 + (-3)$ is -9.

Q5 Answers will vary but should include only negative numbers.

Q6 Whether adding two negative or two positive numbers, both arrows go the same way, taking the sum farther away from the center of the number line (farther away from zero). The difference is that the arrows go to the right when the numbers are positive but go to the left when they're negative.

Q7 When you add two negative numbers, you cannot get a positive sum. Both numbers take the sum in the negative direction from zero, so the sum must be negative.

Q8 As students model various problems, walk around the room and observe them to make sure they can model any problem they are given.

$7 + (-4) = 3$ $\quad -4 + 7 = 3$

$-6 + 2 = -4$ $\quad 2 + (-6) = -4$

$-3 + 7 = 4$ $\quad 3 + (-7) = -4$

$2 + (-5) = -3$ $\quad -2 + 5 = 3$

Q9 When you add a positive and a negative integer, the number that has the larger absolute value tells you whether the answer will be positive or negative. In other words, the sign of the result is the same as the sign of the longer arrow.

EXPLORE MORE

Q10 Each student will model different problems. In every case, the two numbers must be opposites, so that their arrows are the same length but point in opposite directions.

Q11 The order does not matter when you add two numbers. The arrows determine how far you go and in which direction, and it doesn't matter if you follow the first arrow and then the second, or if you follow the second arrow and then the first.

WHOLE-CLASS PRESENTATION

Start the whole-class presentation by animating the addition of two positive integers (Q1–Q3 of the activity). Open the sketch **Adding Integers Present.gsp** and press the step-by-step buttons one at a time, pausing between animations. Ask students to describe what they see as the animation

progresses, and be sure to get observations from several different students. Press the *Reset* button, change the problem by dragging both circles (while leaving the numbers positive), and press the step-by-step buttons again.

Next animate the addition of two negative numbers (Q4–Q7 of the activity). Press *Reset,* drag the numbers so they are both negative, and ask students to predict what will happen now. Use the step-by-step buttons to test their conjectures. Without resetting, ask questions Q6 and Q7, and experiment by dragging to change the values of the numbers.

When students are satisfied with the results of adding two negative numbers, animate the addition of numbers with different signs. Reset again and drag the circles so one of the numbers is positive and one is negative. Ask students to predict how the arrows will behave. (Try to get students to concentrate on the behavior of the model rather than on the numeric answer.) Use the buttons again to show the behavior. Model several more problems (such as those in Q8) involving a positive and a negative number.

Finish the class discussion using Q9, Q10, and Q11. When students propose an answer to one of these questions, have them manipulate the sketch to show why their answer makes sense.

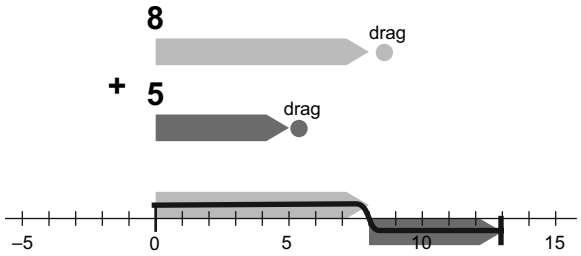
Adding Integers

For GSP5

In this activity you'll add integers using an animated Sketchpad model.

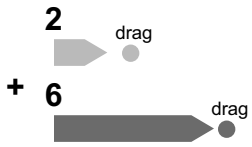
INVESTIGATE

1. Open **Adding Integers.gsp.** This sketch models the addition problem 8 + 5.
2. Press the *Present All* button to see the model in action.



Q1 How does the final position of the arrows show the answer for 8 + 5?

3. Press the *Reset* button, and then drag the circles to model 2 + 6.
4. This time, show the animation step by step: Press the *Show Steps* button, and then press each numbered button in order.

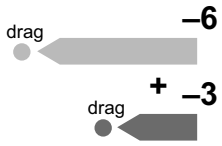


For each problem, press the buttons to show the result.

Q2 Drag the circles and press the buttons to model two other addition problems using only positive integers. Record each problem and the result.

Q3 How do the two upper arrows in the sketch relate to the two lower arrows?

Q4 Model −6 + (−3). What's the sum?

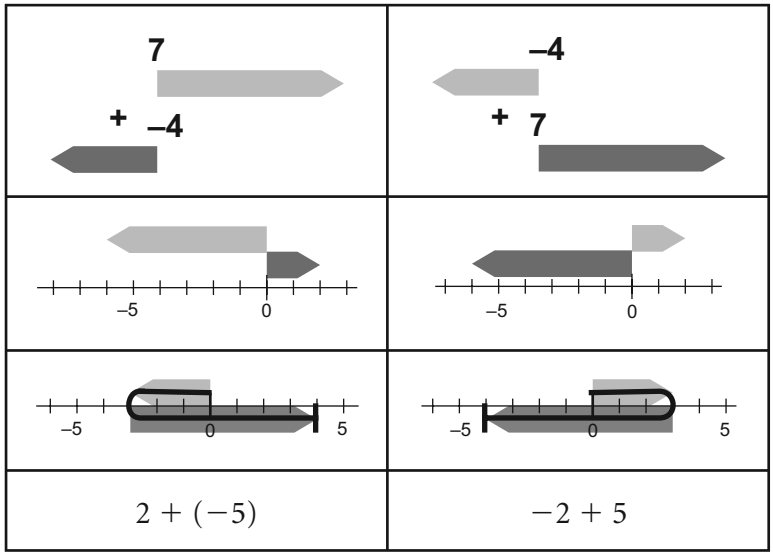


Q5 Model two more addition problems using negative integers. Record each problem and its result.

Q6 How is adding two negative numbers similar to adding two positive numbers? How is it different?

Q7 Can you add two negative numbers and get a positive sum? Explain.

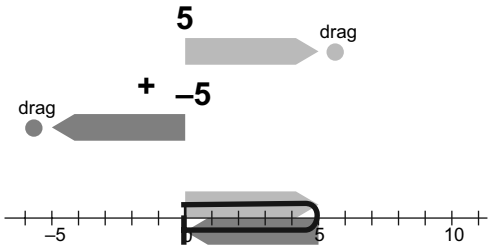
Q8 Model the following eight problems. Record each problem and its answer.



Q9 When you add a positive and a negative integer, how can you look at the numbers and tell whether the answer will be positive or negative?

EXPLORE MORE

Q10 Model four problems for which the sum is zero. Make the first number positive in two problems and negative in two problems. Write down the problems you used. What must be true about two numbers if their sum is zero?



Q11 When you add two numbers, does the order matter? In other words, is $-3 + 5$ the same as $5 + (-3)$? Using the sketch, explain why your answer makes sense.

Use this activity as an introduction to integer subtraction for pre-algebra students, as a start-of-the-year refresher for Algebra 1 students, or as a supplemental activity for any student having difficulty with the topic. It's important for students to have a mental image of operations on integers. Even strong students who rely on verbal rules make careless mistakes that could be avoided by having an internalized picture.

The picture of subtraction presented here is a geometric model in which each number is represented by a vector. (The activity calls them *arrows* because students may not be familiar with the term *vector.*) Vectors incorporate both magnitude and direction (representing the absolute value and the sign of the integer), so practice with this model helps students understand how the signs of the operands come into play.

The questions are critical in encouraging students to internalize the model presented in this activity. Make sure students write clear and detailed explanations (and use complete sentences) when they answer the questions; the extra time it takes them to do so is time well spent.

If there's time and you have a presentation computer with a projector, have different students use Sketchpad to demonstrate to the class their observations or the problems they made up. It's a big help to students if they can listen to, evaluate, and discuss the descriptions and conclusions of their classmates.

INVESTIGATE

These notes sometimes use the terms *minuend* (first number) and *subtrahend* (second number), but these terms are not used in the student material. If you do use them with students, be sure to explain them carefully.

The concept of *additive inverse* is not named, but it plays a prominent role in the animation. You should discuss with the class why the second number must be flipped, even if you don't give a name to that operation.

Q1 During the animation, the arrow for 5 flips from the right to the left. This shows which way the second arrow must go in order to subtract it from the first.

Q2 In their final positions, the flipped second arrow starts from where the first arrow ends, and the answer (3) is at the end of the second arrow. Encourage students to be detailed and specific in their answer to this question.

Q3 Answers will vary. Students should describe the arrow flipping from right to left; encourage them to explain in their own words why it needs to flip in order to do subtraction.

Q4 Answers will vary but should include only problems in which a positive minuend is smaller than a positive subtrahend.

Q5 If both numbers are positive, the result will be positive if the first number (minuend) is larger, and negative if the second number (subtrahend) is larger.

Q6 Some students will record direct observations, and others will interpret those observations. Typical answers will be similar to the following.

Observation: In this problem, $4 - (-3)$, the second arrow starts out pointing to the left, so when it flips it turns around and points to the right.

Interpretation: The second number starts out negative, so when it flips it becomes positive.

Q7 The problems students create will vary. Because the first number is positive and the second negative, the models have in common that, after flipping, both arrows point to the right, and the result must be positive.

Q8 Problems will vary. Because the first number is negative and the second positive, after flipping, both arrows point to the left, and the result is negative.

Q9 As students model various problems, walk around the room and observe them to make sure they can model any problem they are given.

$7 - (-4) = 11$ $\quad -4 - 7 = -11$

$-6 - (-2) = -4$ $\quad -3 - (-6) = 3$

$-3 - 8 = -11$ $\quad -3 - (-8) = 5$

$2 - (-7) = 9$ $\quad -2 - 7 = -9$

Q10 Written as addition problems, these problems become

$7 + 4 = 11$ $\quad -4 + (-7) = -11$

$-6 + 2 = -4$ $\quad -3 + 6 = 3$

$-3 + (-8) = -11$ $\quad -3 + 8 = 5$

$2 + 7 = 9$ $\quad -2 + (-7) = -9$

In each case, to subtract you can change the sign of the second number and add them. This is similar to the way the second arrow flips before the animation shows the answer.

EXPLORE MORE

Q11 For a subtraction problem to have an answer of zero, the two numbers being subtracted must be the same.

Q12 To make the difference the same as the first number, the second number must be zero.

Q13 To make the difference the same as the second number, the first number must be twice as big as the second. For instance, $6 - 3 = 3$, and $-8 - (-4) = -4$.

Q14 The order does matter when you subtract numbers, because only the second arrow is flipped. More sophisticated students will observe that the order matters only if the second number is nonzero, because flipping zero has no effect.

WHOLE-CLASS PRESENTATION

Start the whole-class presentation by animating the subtraction of two positive integers (Q1–Q5 of the activity). Open the sketch **Subtracting Integers Present.gsp** and press the step-by-step buttons one at a time, pausing between animations. Ask students to describe what they see as the animation progresses, and be sure to get observations from several different students. Press the *Reset* button, change the problem by dragging both circles (while leaving the numbers positive), and press the step-by-step buttons again. Pay special attention to Q3 and Q5.

Next animate subtraction problems in which the first number is positive and the second number is negative (Q6–Q7 of the activity). Press *Reset,* make the first number positive and the second negative, and ask students to predict what will happen now. Test their conjectures using the step-by-step buttons. Repeat for several more problems.

Animate subtraction problems like those in Q8 and Q9, and record the answers for each of the problems in Q9. Ask students what patterns they see, and how they could predict the answer from the two numbers being subtracted.

For Q10, ask students to make an addition problem for each of the problems from Q9, and test their addition problems using page 2 of the sketch. Switching back and forth between page 1 and page 2 will reinforce for students the idea of using addition to rewrite a subtraction problem.

Continue the class discussion with as many of the Explore More questions (Q11–Q14) as are appropriate for the class and the available time.

Finish by having students summarize in their own words the relationship between subtraction and addition.

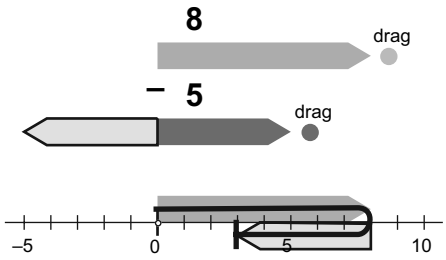
Subtracting Integers

For GSP5

In this activity you'll subtract integers using an animated Sketchpad model.

INVESTIGATE

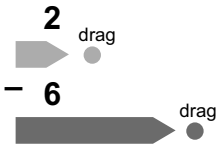
1. Open **Subtracting Integers.gsp.** The sketch models the subtraction problem $8 - 5$.
2. Press the *Present All* button to see the model in action.



Q1 During the animation, what happens to the arrow for 5?

Q2 How does the final position of the bottom arrows show the answer for this subtraction problem?

3. Press the *Reset* button, and then drag the circles to model $2 - 6$.



4. This time, show the animation step by step: Press the *Show Steps* button, and then press each numbered button in order.

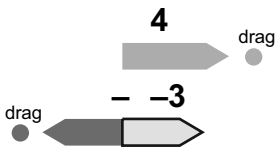
Q3 Describe in your own words what the *3. Make Inverse* step does.

For each problem, press the buttons to show the result.

Q4 Drag the circles to model two more subtraction problems that use positive integers but have a negative result. Record each problem and its result.

Q5 If both numbers in a subtraction problem are positive, how can you tell if the answer will be positive or negative?

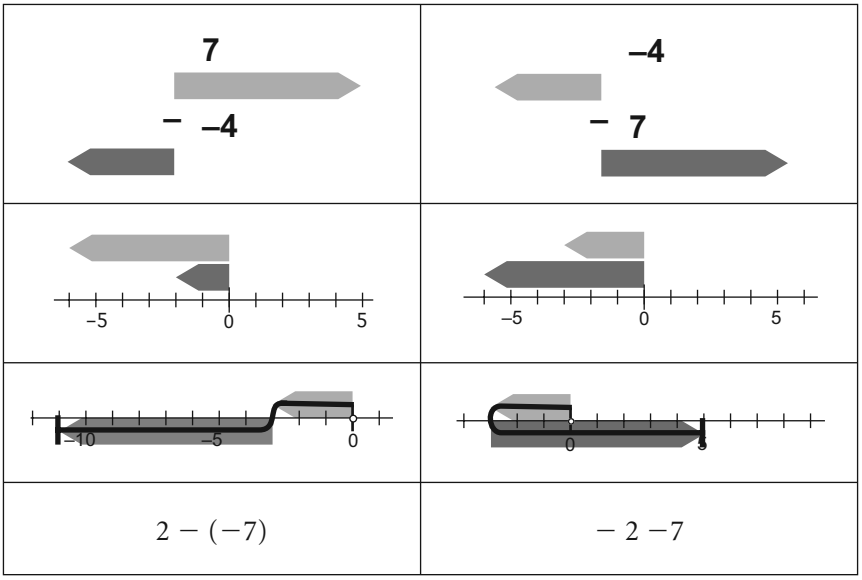
Q6 Model $4 - (-3)$. What's different about the *3. Make Inverse* step this time?



Q7 Model two more problems in which the first number is positive and the second number is negative. Record each problem. What do these models have in common?

Q8 Model three problems in which the first number is negative and the second number is positive. Record each problem. What do these models have in common?

Q9 Model the following eight problems. Record each problem and its answer.

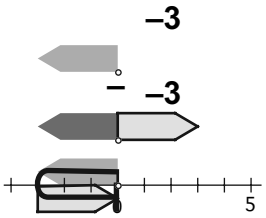


For instance, $7 - (-4) = 11$, so fill in the blank: $7 + \_\_\_ = 11$.

Q10 For each subtraction problem above, write an addition problem that has the same first number and the same answer. What do you notice?

EXPLORE MORE

Q11 Model four subtraction problems for which the difference is zero. Make the first number positive in two problems and negative in two problems. Write down the problems you used. What must be true about two numbers if their difference is zero?



Q12 Model four subtraction problems in which the difference is the same as the first number. What must be true of these problems?

Q13 Model four subtraction problems in which the difference is the same as the second number. What must be true of these problems?

Q14 When you subtract two numbers, does the order matter? In other words, is $-3 - (-5)$ the same as $-5 - (-3)$? Explain in terms of the model why your answer makes sense.