**Objective:** Students explore dynagraphs, an alternative to Cartesian graphs, to develop a feel for various types of functional relationships.

**Student Audience:** Algebra 1/Algebra 2

**Prerequisites:** Students should have some idea what is meant by a function.

**Sketchpad Level:** Easy. Students manipulate a pre-made sketch.

**Activity Time:** 30–40 minutes. Be sure to give students enough time to write detailed and precise descriptions of the dynagraphs (Q3 and Q4). To reduce the amount of time required, students could skip Q4 or the Explore More section.

**Setting:** Paired/Individual Activity (use **Introducing Dynagraphs.gsp)** or Whole-Class Presentation (use **Introducing Dynagraphs Present.gsp**)

The term *dynagraph* was coined by Paul Goldenberg, Philip Lewis, and James O'Keefe in their study "Dynamic Representation and the Development of a Process Understanding of Functions" published by Education Development Center, Inc., and supported in part by a grant from the National Science Foundation.

The motivation for developing and using dynagraphs comes from the often-noted difficulty students have in seeing the graphs of functions as dynamic representations of functional relationships between two quantities and not just as static pictures. By decoupling the input and output axes, and having a segment connect points on parallel axes, students are better able to see the input-output machine view of functions expressed graphically. Being able to drag the input marker gives students the further advantage of actually varying the independent variable and seeing the function as a *dynamic* relationship between input and output.

Dynagraphs can serve as a bridge between the input-output machine model with which students are often introduced to functions and function graphs in the Cartesian plane.

## SKETCH AND INVESTIGATE

**Q1** Answers will vary, but should basically describe functions as consistent input-output machines. In other words, they are relations or mappings between input values and output values such that any valid input value maps to a single output value.

**Q2** The dynagraphs do represent functions because they map input values to output values and they are consistent—a particular input value will always point to the same output value.

**Q3** Answers will vary, but should not involve numbers or formulas. Good answers will in general include dynamic descriptions ("As the input is dragged steadily from left to right, the output …") and note any symmetries present.

## NUMBERS, NUMBERS, NUMBERS

**Q4** See Q3.

**Q5**

| | |
|---|---|
| a. $t(1) = -1$ | b. $t(5) = 7$ |
| c. $x = -1$ | d. $g = -6$ |
| e. $p = -6$ | f. $m = \ldots -7, -3, 1, 5, \ldots$ |
| g. $v(4) = 2$ | h. $v(-4)$ is undefined |
| i. $r = 9$ | j. $z = 1$ |
| k. $s = 3$ | l. $a = 2.5$ |

## EXPLORE MORE

**Q6** Function *j* has an absolute minimum of 0 at 0. Function *u* has an absolute maximum of 6 at (…, −7, −3, 1, 5, …), and has an absolute minimum of −6 at (…, −5, −1, 3, 7, …). Function *v* has an absolute minimum of 0 at 0.

**Q7** p3: $t(1) = -1$; $t(5) = 7$; $x = -1$
p4: $g = -6$; $p = -6$; $m = 1 + 4k$ for integer values of $k$
p5: $v(4) = 2$; $v(-4)$ is undefined; $r = 9$
p6: $z = 1$; $s = 3$; $a = 2.5$

**Q8** Answers vary.

## WHOLE-CLASS PRESENTATION

Use this sketch to explore this highly dynamic visual representation of functions with your students. Dynagraphs differ from Cartesian graphs in that you can make the variables really vary, so emphasize the variation by different students drag the independent variable for each function. Students can also press Animation buttons to leave variables moving on the screen.

This whole-class presentation allows students to gain a dynamic perspective on the notion of function and emphasizes the way in which the variables really vary.

## SKETCH AND INVESTIGATE

**Q1** Begin by asking students to describe a function in their own words. Get responses from several students, and encourage a diversity of descriptions. Consider forming small groups of two or three students and asking each group to create its own written description, suitable for explaining functions to someone who isn't familiar with them.

1. Open geometricfunctions.org/links/introducing-dynagraphs. Four dynagraphs appear, each in a different color.
2. Explain that the input and output markers represent the variables and that this model allows you to vary the variables by dragging the markers. Use the **Arrow** tool to drag input marker *a*. After dragging it a bit, use the *Animate a* button to leave it in motion.

**Q2** Ask students whether the behavior they observe represents a function, based on their description of what a function is. Solicit different explanations from as many students as possible.

**Q3** Ask students to describe the behavior of this first function. They will want to call the tick mark "zero" or "the origin," and they will want to describe movement to the left or right as "increasing" or "decreasing." These characterizations are based on numbers; resist them, and instead encourage students to describe the behavior in terms of position, movement, and symmetry. Consider asking students whether the function has a "fixed point"—a state in which the input marker and the output marker are at exactly the same position.

Leave each function in motion while you drag and discuss the remaining functions.

**Q4** Drag the input marker for the second function and have students observe. Leave it in motion while students describe the behavior. (Students often want to call this function a "constant function." Rather than describing this answer as wrong, ask them whether it's the output that is constant or whether there is something else about this function that they view as being "constant.")

**Q5** Use the *Animate c* button to put the third function's input marker into motion. (Students will often laugh at this function, and you may want to ask them how often they have laughed at a mathematical function.) Have them describe this function in detail. They may want to give it a name.

3. The remaining pages show dynagraphs with numbers added to the axes. Have students answer the questions on each page, and then allow them to see the algebraic formulas underlying the behavior of the dynagraphs.

 